I, You and *It:* Pronouns and Students' Understanding of Introductory Algebra

Judith Falle
University of New England
<jfalle@une.edu.au>

Some studies suggest that teachers make inferential judgements about students' understanding based on the ways in which they use language. One aspect of language use is that of pronouns. Students' use of first, second, or third person pronouns indicates the extent of commitment to the truth-value of their utterances. This paper discusses a study that investigated students' use of pronouns as they explained their understanding of introductory algebraic expressions and equations. The evidence suggested that pronoun use can act as an indicator of the level of students' conceptual development in this context, which might also be aligned with a theoretical model.

Teachers listening to students, or reading their work, make judgements about their understanding based on the way students explain their thinking. Elements of that judgement include not only the students' use of appropriate (or inappropriate) mathematical terminology, but also more subtle features that operate in any discursive environment.

Studies by Bills and Gray (2001) and Bills (2002) investigated the language used by primary students discussing their arithmetic understanding. Boero, Douek, and Ferrari, (2002), conducted studies into the written mathematical explanations of students entering university science and computing courses. Their findings indicated that the quality of students' understanding of mathematical ideas might be inferred from ways in which they use the language. Rowland (2002), in a broad study of students' language use, identified linguistic features such as modality, pronouns, and tense that serve to indicate quality of students' understanding. Some of the findings from these studies are summarised in the following.

Explanations given by successful students tend to be propositional in nature – generalised statements that make little or no reference to specific mathematical examples. Such generalisation tends to be marked by the use of the simple present tense and the impersonal *you* (instead of the archaic *one*). Generalisation by students implies a confident understanding of the topic, an ability to see and articulate mathematical similarities between examples, and to see individual examples as representative of a class of mathematical *objects* (Sfard, 2002).

Less-successful students tend to make more specific statements that use the first person pronoun, *I,* and explain their thinking in terms of recounting procedures with particular examples. Particularity implies that students cannot make explicit their perceiving of class characteristics that might be shared by sets of mathematical objects, although such knowledge might be implicit in their responses to several examples.

Rowland (2002) noted that where students struggle with inaccessible concepts, they make vague statements, characterised by the third person pronoun, *it*. Vagueness suggests that students are far less certain of their knowledge; that they are "operating in a conjecturing environment" (p.67). In contrast to generalisation and particularity, vagueness leaves the listener, or reader, uncertain as to the subject of the utterance.

The studies cited above have focused on young students in primary school, and older students of greater mathematical experience entering university. The study outlined in this paper focuses on students in the intervening years, investigating language use by students

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1). Palmerston North, NZ: MERGA.

in the first four years of secondary school, in the particular and often problematic context of introductory algebra. Algebra presents many difficulties to students, at different stages in their mathematical development. Difficulties can be masked by an emphasis on pen-and-paper manipulation of expressions, but might be revealed when students engage in mathematical discourse. Many aspects of language-in-use act together to indicate a student's confidence of understanding. One such aspect is the way in which use of different personal pronouns indicates the degree to which students are secure in their mathematical understanding. This paper discusses students' use of pronouns with respect to their overall success on algebra questions and the difficulty of those questions.

The Study

Investigation into students' use of pronouns was part of a larger study into students' language use, in the context of introductory algebra. Students (N=222) in Years 8 and 9, from three secondary schools in regional NSW, completed a test (referred to as a survey) consisting of 40 symbolic algebra items. Responses to the survey were marked as either correct or incorrect, and Rasch-modelled (using Quest software, Adams & Khoo, 1996)[1]. The model provided objective measures of student ability and item difficulty, on which analyses of language patterns were based.

From the model, three groups of students were identified, representing three levels of success on the survey (ability groups). The range of ability estimates for each group were: -4.89 to -1.28 logits (Low–ability group); -1.11 to 0.93 logits (Average-ability group); and, 1.1 to 4.93 logits (High-ability group). Two sub-groups were identified in the High-ability group, with ability estimates of 1.27 to 1.65 logits (the High-7 sub-group), and from 2.58 to 3.67 logits (High-3 sub-group).

Three clusters of survey items were also identified, representing three distinct clusters of item difficulty. The range of difficulty estimates for the clusters were: Cluster 1 (the easiest items) -3.27 to -1.98 logits; Cluster 2 (items of middling difficulty) -1.17 to 0.46 logits; and, Cluster 3 (the most difficult items) 1.18 to 3.56 logits. The means for these clusters approximated the means for each of the corresponding the student-ability groups.

The students interviewed (n = 31) were drawn from each of the three ability groups identified from the model. These were students who agreed to be interviewed, and who had the requisite permissions from schools and caregivers.

Interviews were *contingent* (Rowland, 2002): the interview protocol consisted of a single question asking students to explain their thinking as they responded to items in a set of items, and the interviewer was free to ask further questions for clarification of the student's thinking. Interviews were conducted with individual students and lasted from 30 to 60 minutes, audio-taped and transcribed for later analysis.

Analysis of Pronoun Use

The proportion of personal pronouns used by students in their conversations was small. Throughout the interviews, where students were asked to respond to sets of items as a whole there were many instances where students either declined to make any mathematical response, or addressed a single item only. Analysis of the frequency of personal pronouns

[1] Rasch modelling calculates probabilistic estimates of item difficulty and student ability (success on the test items), and maps both estimates on the same equal interval scale. Estimates are in *logits*.

in responses by students in each of the groups to specific items was found to give a clearer pattern of changes in linguistic behaviour to items of different difficulties. Hence, students' use of the pronouns *I*, *you* and *it* was examined in the particular context of responses to items in Sets 5 and 6.

These sets consisted of equations to be solved (Table1), and were the sets that produced the longest conversations, particularly Set 5. This was due, in part, to there being more items to discuss in Set 5, than in any of the other sets of items. Conversations were also longer because the students, on the whole, appeared to be most comfortable with these types of items. The two sets of equations were chosen also because the students' responses tended to be directed to the individual items, and most students attempted to respond to all items in the two sets.

Table 1
Survey Items (Equations) from Interview Sets 5 and 6

	Item number	Item	Item difficulty estimates (logits)
Set 5 (mean difficulty: -0.09 logits)	27	$x + 5 = 7$	-3.1
	28	$4y = 20$	-2.3
	29	$2t - 23 = 49$	-0.5
	31	$4(p + 3) = 32$	-0.1
	32	$10y = 5$	-0.3
	34	$x + (x + 2) = (x - 1) + 8$	1.51
	37	$5a - 4 = 2a + 8$	1.22
	39	$ax = 5$	3.09
Set 6 (mean difficulty: 0.34 logits)	30	$x/4 = 12$	-0.9
	33	$(x + 3)/2 = 7$	-0.6
	36	$63/x = 180$	2.64

The mean difficulty of Set 5 meant that it was the third easiest set, (in terms of the Rasch model) and near both the constrained mean difficulty (0.0 logits) of all items in the algebra survey and the mean ability estimate for the Average-ability group (-0.01 logits). Set 6, also consisting of equations, was one of the more difficult sets. The two sets provided a range of data that allowed the examination of the use of personal pronouns. Students' responses were analysed firstly with respect to the difficulty of the items, and secondly in terms of responses by each of the ability groups to individual items in Sets 5 and 6.

Personal Pronouns and Changes in Item Difficulty

Table 2 details the frequency of occurrence of the pronouns *I*, *you* or *it* in responses by students (n=31) to items in Sets 5 and 6. Each item is identified by the survey number, together with the Rasch-difficulty estimate in logits (Table 1 above), the number of correct responses to the survey item by the students interviewed, and the number of refusals to respond to the interview question. Each response by a particular student to a particular

Table 2
Frequency of Statements Using Pronouns I, it *or* you *by Students Responding to Items in Set 5 and Set 6**

	Items in Set 5 and Set 6 in order of Rasch difficulty										
Item number	27	28	30	33	29	32	31	37	34	36	39
Rasch difficulty	*-3.1*	*-2.3*	*-0.9*	*-0.6*	*-0.5*	*-0.3*	*-0.1*	*1.22*	*1.51*	*2.64*	*3.09*
	Responses by all ability groups										
No. Correct responses	29	23	21	20	17	19	3	11	8	5	1
No reply	0	1	1	6	4	3	19	9	12	11	5
I	13	13	14	12	12	9	6	14	13	9	8
it	3	2	**11**	6	4	**10**	2	2	3	**7**	1
you	15	15	5	7	11	9	4	6	3	4	7

* Items ordered by Rasch difficulty estimates

item was coded according to the most frequently occurring pronoun in that response. Thus a response might be an "*I*-response", a "*you*-response", or an "*it*-response". (e.g., Item 31, difficulty estimate of 1.22 logits, attracted three correct responses in the survey. During the interviews, 19 students declined to answer, or to attend to the item; six students used *I* as the predominant, or only, personal pronoun in their responses; two most frequently used *it*; and, four students tended to use *you* most frequently in their responses to this item.)

The number of responses tended to decrease as the items became more difficult. Item 31 [4(p+3)=32] was often avoided during the interview, unless the students were asked specifically about the item. On the basis of their earlier responses to the set, many of the less-able students were not asked to respond to Item 39 [ax=5] and, hence, fewer responses of any type were recorded. This item, although one of the most difficult in the survey, attracted confident answers from the Average- and High-ability students who, in most cases, suggested possible values for a and/or x as being factors of five, or 2.5.

The occurrence of the pronoun *you* suggests that the student is, in some ways, making a general, propositional statement. The number of these second-person responses tended to decrease as items became more difficult. Item 29 [2t – 23=49] was the exception.

One of the most able students explained his thinking about items in set 5, using the equation x + 5=7 as an example, and one other equation [t + 48.4=201.9] introduced in order to encourage an algebraic approach rather than trial-and-error.

> You have to get x as the subject, so to get x as the subject you have to take away plus 5 from each side, so you get x equals two, and on the other one, you have to get t as the subject so you have got to get rid of minus 48.4 so you add that to the other side?

The student states a general rule for an appropriate procedure for solving any equation, using the impersonal *you*, and then illustrates the rule using specific examples. At no time during the interview did this particular student use the first or third person pronouns.

In contrast, the use of *I* suggests a degree of uncertainty, the student merely stating what he or she would do in a particular circumstance.

I (Interviewer): Why take the 5 from the 7, when in the next one 48.4 is added?
S: Because in the first place I had to add, so to go, to turn it around I had to minus and with that one I minused in the first place so to turn it around I had to add. Um …

The occurrence of the pronoun *it* indicates vagueness – a position less assured even than that of a student who provides a personal opinion about a specific item by the use of *I*, rather than a general rule.

The following example is a verbatim extract from the interview with a student for whom Item 37 [$5a - 4 = 2a + 8$] was difficult.

Ummm [regarding Item 37, and thinking for a considerable time] For 37 I guess um… I'd try work it out like… Just try figure one out and then see if it equals the other.
… Guess…Like think of a number and then do that [indicating by gestures that she would substitute a number in both sides of the equation] and then see if it's… How much higher or lower, and then try like to get to another number in between. I think there's an easier way but I don't really know.

The student has used the first person, *I*, to describe what she would do. The use of the third person *it* refers to both the item ("I'd try work it out like") and, in the following sentence, to one side of the equation ("…then see if it equals the other"). The use of gestures ["…that…"] to indicate substitution demonstrates a lack of appropriate terminology. The indefinite referent for *it's* ["…and then see if it's…"] could be either the entire equation, or one side of the equation. The uncertainty inferred from the utterance, can be confirmed by the student's final statement.

Items that attracted the greatest number of statements in the first or third person were those that required careful reading, e.g., Item 30 [$x/4=12$], or involved challenging arithmetic, such as Item 32 [$10y=5$], and Item 36 [$63/x=180$]. The two easiest items, Item 27 [$x+5=7$] and Item 28 [$4y=20$], were usually responded to in the first or second person.

In Table 2, the pronoun *it* can be seen to occur most frequently at three points (in bold). These occur at difficulty estimates that are near the thresholds for each ability group. The first occurs at Item 30 (-0.9 logits). This is at a difficulty level slightly above the lower bound for the Average-ability group (-0.94 logits) and well-above that of the Low-ability group (-1.28 logits). The second peak occurs at Item 32 (-0.3 logits), which is greater than the ability estimates for half of the Average-ability group. The third peak occurs at Item 36 (2.64 logits), which is greater by 0.9 logits than the upper ability estimate (1.65 logits) for seven students in the lower part of the High-ability group (High-7) and greater than the lower ability estimate (2.58 logits) for the remaining group of the three students with the highest ability (High-3).

These figures suggest that when students meet items that are beyond their ability level, their statements will be couched in vague terms, indicative of a struggle to make understanding explicit. If the sum of the numbers of first person and third person statements is considered (implying specificity or vagueness), the ability of students to make general statements, inferred from their use of the second person, decreases with the difficulty of the items.

Personal Pronouns and Ability Group Responses to Changes in Item Difficulty

Use of the pronouns *I, you* and *it* was also analysed with respect to the ability groups for each item in Sets 5 and 6. For this analysis, the High-ability group was split into two sub-groups delineated by a gap of 0.9 logits between the three highest ability estimates (High-3) and the remaining seven (High-7) (see above).

Table 3 outlines the frequency of statements consisting of the three personal pronouns by each of the ability groups in their responses to items in Sets 5 and 6. The frequency is expressed as a proportion of responses, calculated as the average number of statements using a particular pronoun (*I*-responses, *You*-responses, *it*-responses) for each student in the group (or sub-group). (e.g., in the Average-ability group, five students out of the fourteen in the group used *I* statements when responding to Item 27, and nine students out of the fourteen responded with *you* statements to that item.) The proportions are expressed as decimals to one decimal place.

Shaded areas in Table 3 indicate where students in the ability groups (or sub-groups) have a less than, or equal, chance of obtaining correct answers to the items. The Average-

Table 3
Proportion of I, you *or* it *Statements by Ability Groups**

Ability groups [Range in logits]	Response	Items in Set 5 and Set 6 in order of Rasch difficulty									
		27	**28**	**30**	**33**	**29**	**32**	**37**	**34**	**36**	**39**
		-3.14	*-2.33*	*-0.93*	*-0.58*	*-0.52*	*-0.34*	*1.22*	*1.51*	*2.64*	*3.09*
HIGH-3 [2.58 to 3.67]	Nil	0	0	0	0	0	0	0	0.3	0	0
	I	0.3	0.3	0.3	0.3	0	0.3	0.3	0.3	0.3	0
	it	0	0	0	0	0	0	0	0	0	0
	you	0.7	0.7	0.7	0.7	1	0.7	0.7	0.3	0.7	1
HIGH-7 [1.27 to 1.65]	Nil	0	0	0	0.1	0.1	0	0.0	0.3	0.3	0
	I	0.6	0.1	0.7	0.6	0.6	0.6	0.7	0.7	0.4	0.3
	it	0	0	0	0	0.1	0.1	0	0	0.3	0.4
	you	0.4	0.9	0.3	0.3	0.1	0.3	0.1	0	0	0.3
AVERAGE [-0.94 to 0.93]	Nil	0	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.3	0.2
	I	0.4	0.5	0.4	0.4	0.4	0.1	0.5	0.5	0.4	0.3
	it	0	0.1	0.1	0.4	0.1	0.4	0.1	0.1	0.3	0.5
	you	0.6	0.3	0.1	0.1	0.4	0.3	0.1	0.1	0.1	0.1
LOW [-2.86 to -1.28]	Nil	0	0	0	0.4	0.1	0.1	0.6	0.7	0.9	0.6
	I	0.4	0.6	0.4	0.3	0.4	0.3	0.1	0	0	0.3
	it	0.4	0	0.6	0	0.1	0.4	0.1	0.3	0.1	0.1
	you	0.1	0.4	0	0.3	0.3	0.1	0.1	0	0	0

*Items are ranged in ascending Rasch order, student ability in descending order

ability group has a wide spread of ability (1.8 logits) either side of the mean item difficulty estimate of 0.0 logits. Items where some students in the group could be expected to struggle, whilst others will not, are indicated by diagonal hatching in columns under Items 33, 29 and 32. There is also a similar transitional area for the Low-ability group at Item 30.

Of the High-3 sub-group, one student persistently used *I*. This seemed to be an individual idiosyncrasy. The others used *you* almost exclusively. One only student in the sub-group avoided responding to Item 34 [$x+(x+2)=(x-1)+8$], and none of the students used the third person pronoun *it*.

Students in the High-7 sub-group tended to use *I* more frequently then *you*, and more frequently than students in the other ability groups. However, they did not tend to use *it*, until responding to the two most difficult items. These distinctively different linguistic behaviours reflect the gap in ability estimates of 0.93 logits between the sub-groups of the High-ability group.

The Average-ability group tended to use *I* and/or *it* most frequently, more frequently than *you*, except for Item 27 [If x+5=7]. *It* began to be used more frequently as the items became more difficult.

The Low-ability group also tended to use *I* and *it* more frequently than *you*, particularly when responding to items within the ability estimate range for the group (the least difficult equations, Items 27, 28 and 30). This group increasingly gave "nil" responses the more difficult the items became.

Discussion

When ability groups were analysed for the use of the pronouns *I* or *you,* the differences were most marked when the most difficult sets of items were considered (items 37, 34, 36 and 39). In general: students in the High-ability group tended to use the pronouns *you* and *I*; those in the Average-ability group used *I* and *it*; and students in the Low-ability group also tended to use *I* and *it* more frequently than you,

Taken as a whole, students in the High-ability group (combined High-3 and High-7) tended to use *I* less frequently, and to use *you* more often, thus suggesting a security of knowledge. However, there was a difference in this frequency when the sub-groups were considered.

The sub-group High-3 encountered few items with which they would be insecure. None of these students was vague (as indicated by the use of *it)*, and two of the three students consistently used *you*. This was a small group, and therefore the use of *I* consistently by the third student gives greater weight to the occurrence of this pronoun than might occur with a larger sample.

The High-7 sub-group appeared to make a sharply defined shift from the use of *you* to the use of *I* when the difficulty estimates of items neared the threshold for this group (1.22 logits). Students in this sub-group also began to use *it* when dealing with items 0.8 logits above the sub-group's ability threshold. Such a shift in frequency of use suggests a connection between the ability of students to respond to items on the basis of general or principled understanding and their use of *you*. On the other hand, the use of *I* signalled students' insecurity of knowledge as items posed greater conceptual challenges (i.e., item difficulty estimates were higher than students' ability estimates).

When students responded to items that were of a difficulty estimate lower than their ability, they tended to use the pronoun *you*. When item difficulty approached or became greater than the students' ability estimates, the use of *I* and/or *it* became more frequent. The use of *I* and *it* occurred most frequently with the students in the Low-ability group. In particular, the use of *it* was often in association with general, rule-like statements. At one level of analysis these statements bore close resemblance to the statements made by the students in the High-ability group. The use of the pronoun *it*, instead of the general *you*, indicates that these statements were vague utterances rather than statements of firmly understood general principles.

Students in the Average-ability group tended to use *I* more often than expected. This could be associated with these students attempting to answer each item in a set individually, rather than addressing them as a group of items sharing common

characteristics. These students appear to be unable to make any general propositional statements about items in a set. This pattern could be seen clearly when responses by the Average-ability group to items in Sets 5 and 6 were analysed. Despite the difficulty estimates of some items being below the threshold of ability estimates for the group, *I* was used approximately 50% of the time. This frequency did not change markedly. Its use suggests that students in this group have a "singular" perspective, reflected by their tendency to focus on one item at a time in each set.

The change in frequency of use of personal pronouns by students indicated a change in their security of knowledge. A shift from the general *you* to the more personal *I,* to the use of the vague *it,* occurred as items became more difficult. These shifts occurred earlier with the Low-ability group, and later with the High-ability group, reflecting the capacity of students in the groups to deal with items of increasing difficulty. These data for pronoun use, and item types and difficulty, can be mapped onto a theoretical framework, such as the SOLO model, demonstrating a parallel developmental sequence of language use and conceptual change.

Pronoun use by students is not an exclusive indicator of the quality of their understanding of a particular mathematical idea, but when taken together with other linguistic features, can provide a measure of that understanding. As it is, when teachers respond to student utterances in an intuitive way, these data suggest that they are responding to language use in the same ways that they would respond during normal conversations. If, as suggested by several authors (e.g. Boero, Douek & Ferrari, 2002; MacGregor & Price, 1999; Tall, draft) that language development – the acquisition of a mathematical register – develops along with mathematical understanding, then indicators such as pronoun use reflect that development, alerting teachers to students whose understanding is fragile, as well as to those who appear to have a sound grasp of the ideas.

References

Adams, R.J. & Khoo, S. (1996). *Quest: The interactive test analysis system, Version 2.1*. [Computer software]. Melbourne: ACER.

Bills, C. (2002). Linguistic pointers in young children's descriptions of mental calculation. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education*, (pp. 97-104). Norwich: PME.

Bills, C. & Gray, E. (2001). The "particular", "generic" and "general" in young children's mental calculations. In M. van Heuval-Panhuizen (Ed.) *Proceedings of the 25th Annual Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 1, pp.153-160). Utrecht, The Netherlands: PME.

Boero, P., Douek, N., & Ferrari, P. (2002). Developing mastery of natural language: Approaches to theoretical aspects of mathematics. In L. English (Ed.), *Handbook of International Research in Mathematics Education* (pp.241-267). London: Lawrence Erlbaum.

MacGregor, M. & Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, 30(4), 449 – 467.

Rowland, T. (2000). *The Pragmatics of Mathematics Education: Vagueness in Mathematical Discourse*. London: Falmer Press.

Sfard, A. (2002). Thinking in Metaphors and Metaphors for Thinking. In D. Tall & M. Thomas (Eds), *Intelligence, Learning and Understanding in Mathematics*. (pp.79-96). Flaxton: Post Pressed.

Tall, D. (draft). Three Worlds of Mathematics and the Role of Language. Chapter 6 draft in *Mathematical Growth: From Child to Mathematician; Journeys through three worlds of mathematics*. Retrieved 24 February, 2009 from http://www.davidtall.com/mathematical-growth